Spectral Graph Theory, 2021/22 Class

Lecturers: Dr Ahmad Abdi, Dr Neil Olver

Meeting times: Lectures are held **Friday, 10-12** (weeks 1-10). Seminars are held **Wednesday, 10-11** (weeks 2-11). Starting from week 4, the seminar slot has been widened to **Wednesday, 10-11:30**.

Zoom link for week 1 (and other online meetings): click here.

Classrooms for weeks 2-11: Lectures are held in **MAR.2.09**, and seminars in **32L.LG.10**.

Slack channel: Our discussion forum will be on Slack. Join here (the link expires Feb 13, 2022).

Description

Spectral graph theory is concerned with how combinatorial properties of graphs relate to the algebraic structure of certain matrices associated with the graph. One can look at the adjacency matrix of an undirected graph, which is a symmetric matrix, and consider the list of its eigenvalues, called the *spectrum*, along with the corresponding eigenvectors. The spectrum gives us important insight about the graph and its induced subgraphs, and perhaps surprisingly, this viewpoint can be used in the design of graph algorithms, such as network flow problems, plane drawings of planar graphs, clustering, isomorphism testing, etc.

The course will consist of hybrid lectures and seminars on Zoom and in person. During the seminars, we will discuss the course material as well as the homework assignments.

Prerequisites

The course is offered to both MSc and PhD students. Basics of linear algebra and graph theory will be assumed. Some background reading on linear algebra will be provided before the course starts, since a refresher may be helpful.

Outline

The course will begin with a rigorous but accessible treatment of core topics in spectral graph theory.

- Eigenvalue interlacing, and applications to graph theory
- The Laplacian, the matrix tree theorem, electrical flows, Kirchhoff's effective resistance theorem, Transfer-Current Theorem
- Connections to electrical networks, random spanning trees, and random walks
- Spring layout drawings of graphs using spectral methods
- Clustering: how to find good ways of partitioning a graph into pieces via the spectrum
- Expander graphs: sparse graphs with exceptional connectivity properties

The latter part of the course will build on the basics and cover some more advanced topics. *These topics are subject to change.*

Assessment

Formative assessment: There will be 5 homework assignments each of which will be marked and the student will receive feedback.

Discussing with your fellow students is allowed, but you *must* state on your assignment everyone you discussed with, as well as any references you used (aside from the references we have pointed you to). Your solutions must, however, be written completely alone.

Summative component: For MSc students there will be a 3h final exam (%100) in ST. For PhD students there will be a final project (%100) in ST.

References

Lecture notes will be provided. The following references will also be useful.

- Spectral and algebraic graph theory (draft) by Daniel Spielman. This book is quite accessible, but (being a draft) somewhat rough in places.
- Algebraic graph theory (Springer, 2001) by Chris Godsil and Gordon Royle. This book gives a rigorous treatment of fundamental notions in Algebraic Graph Theory.